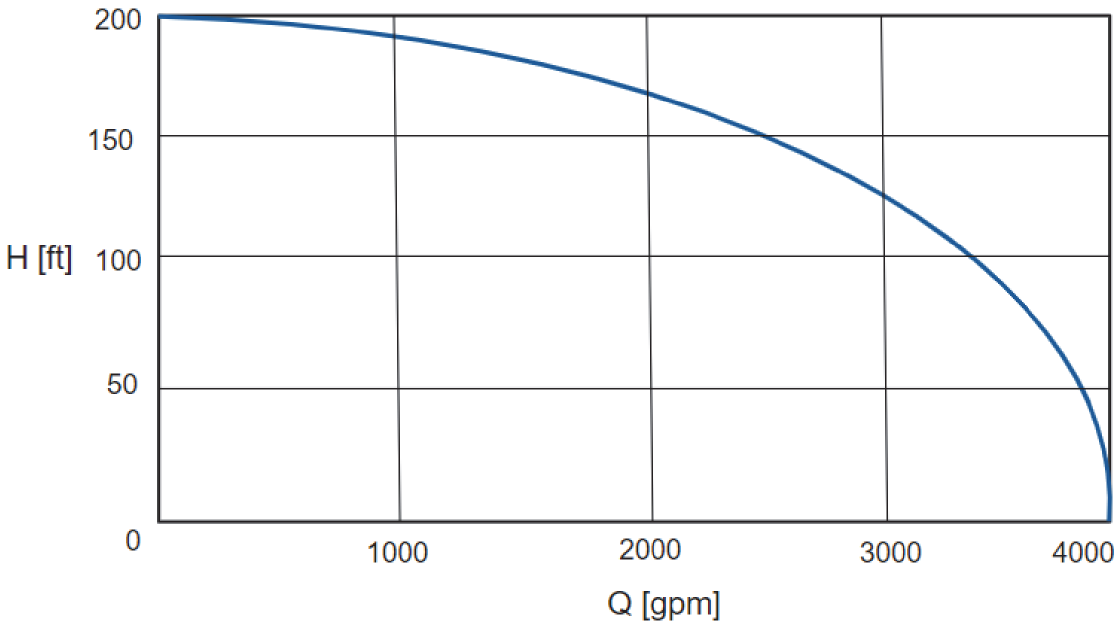
33.50 **A centrifugal pump with the below pump curve transfers water through a nominal $12in$ pipe from an open reservoir at sea level to another open reservoir at $100ft$ elevation one mile away. The friction factor is 0.018 and there are (4) 90-degree elbows in the piping distribution. What is the volume flow rate?**



A. $1800gpm$

B. $2200gpm$

C. $2800gpm$

D. $3200gpm$

Use the modified Bernoulli Equation for head added by a pump.

$$h_A = \frac{P_2 - P_1}{\gamma} + \frac{v_2^2 - v_1^2}{2g} + z_2 - z_1 + h_f$$

Neglect the velocity term. The differential static pressure is zero since the water is being pumped from one open reservoir to another. The difference in height is $100ft$. The friction losses, h_f, are a function of the equivalent length of pipe which includes 1 mile of physical piping plus additional equivalent length for the elbows. Refer to the Equivalent Lengths for Elbows table and assume a typical value for the velocity of $\sim 5fps$. The elbows are only minor losses and do not have a dramatic impact on the total equivalent length.

$$L_{eq} = 5280ft + (4)(29ft) = 5396ft$$

Write an expression for h_f as a function of the velocity using the Darcy-Weisbach Equation. Take the diameter as exactly $1ft$ to simplify the calculation slightly.

$$h_f = \frac{fLv^2}{2Dg}$$

$$h_f = \frac{(0.018)(5396ft)v^2}{2(1ft)\left(32.2\frac{ft}{s^2}\right)} = 1.51v^2$$

Replace v^2 such that h_f becomes a function of the volume flow rate, Q.

$$Q = vA$$

$$v = \frac{Q}{A}$$

$$v^2 = \frac{Q^2}{A^2} = \frac{Q^2}{\left[\frac{\pi}{4}(1ft)^2\right]^2} = 1.62Q^2$$

Substitute into the expression for h_f.

$$h_f = 1.51\left(1.62Q^2\right) = 2.45Q^2$$

Substitute into the equation for the total head added by the pump, h_A. Note Q must have units of $\frac{ft^3}{s}$ in this equation for consistency.

$$h_A = 100ft + 2.45Q^2$$

Create the system curve in tabular form. For organizing the calculation, create a table showing each step, starting with Q in gpm, followed by Q in $\frac{ft^3}{s}$,followed by Q^2, then $2.45Q^2$,then h_A. Compare the ordered pairs of $Q_{[gpm]}$ and h_A to the pump curve in the problem statement and find the intersection. This is the operating point and indicates the volume flow rate and head for the conditions given.

$Q_{[gpm]}$	$Q_{\left[\frac{ft^3}{s}\right]}$	Q^2	$2.45Q^2$	$h_{A,[ft]}$
500	1.11	1.2	3	103
1000	2.22	4.9	12	112
1500	3.33	11.1	27	127
2000	4.44	19.8	48	148
2500	5.55	30.9	76	176
3000	6.66	44.4	109	209
3500	7.77	60.5	148	248

By visual inspection, observe that the system curve will intersect the pump curve between $2000gpm$ and $2500gpm$, at approximately $\sim 2200gpm$. Note the final velocity turns out the be around $6fps$, therefore the assumption for the equivalent length of the elbows was reasonably close.

Answer B